An approximation to an efficient and executable fundamental physics curriculum

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1 Introduction

1.1 Question

- What is an approximation to a theoretical physics curriculum that is co-optimized between the goals of being both comprehensive (up to the fringe of existing experimental verification, but not necessarily quite rigorous enough to prepare for original research) and efficient, assuming one already has a background in linear algebra, differential equations, and related mathematics?
  - Before reviewing the proposed curriculum it would be reasonable to consider the much broader and more comprehensive perspective from Gerard 't Hooft: How to become a GOOD Theoretical Physicist.

1.2 Local and global perspectives

- I think it is helpful to be aware of the following perspectives before getting started
  - A quote regarding the relationship between the local and global structure of theoretical physics
    - "All of physics has two aspects: a local or even infinitesimal aspect, and a global aspect. Much of the standard lore deals just with the local and infinitesimal aspects – the **perturbative** aspects and fiber bundles play little role there. But they are the

all-important structure that governs the global – the **non-perturbative** – aspect. Bundles are the **global** structure of physical fields and they are irrelevant only for the crude local and perturbative description of reality." [Note: Of course... the latter should not be viewed as a **criticism** as those are the components that support concrete calculations that make direct contact with the analysis of experiments]
–Urs Schreiber's response to physics.stackexchange question: Intuitively, why are bundles so important in Physics?

- A paragraph written by Edward Witten [17] on 'Physics and geometry' p. 280 in the 1986 Proceedings of the International Congress of Mathematicians (and p. 20 in this version) with suggested updates from Eric Weinstein's The Portal 'Graph, Wall, Tome project'
  * If one wants to summarize our knowledge of physics in the briefest possible terms, there are really three fundamental observations
    1. Space-time is a pseudo-Riemannian manifold, M, endowed with a metric tensor and governed by geometrical laws
    2. Over M is a principal G-bundle, P_G, with a non-Abelian structure group G
    3. Fermions are sections of $(\hat{S}_+ \otimes V_R) \oplus (\hat{S}_- \otimes V_{\tilde{R}})$. R and $\tilde{R}$ are not isomorphic, but should be complex linear representations of G
  * See here for the definitions of S, V and R.

  All of this must be supplemented with the understanding that the geometrical laws obeyed by the metric tensor, the gauge fields, and the fermions are to be interpreted in quantum mechanical terms.

For reference, the following definitions are taken from Simon Rea's notes on Frederic Schuller's Lectures on the Geometrical Anatomy of Theoretical Physics. Note that the notation above and below are not equivalent.

Definition. A *bundle* (of topological manifolds) is a triple (E, π, M) where E and M are topological manifolds called the *total space* and the *base space* respectively, and π is a continuous, surjective map $\pi\colon E \to M$ called the *projection map*.

Definition. Let $E \xrightarrow{\pi} M$ be a bundle and let F be a manifold. Then, $E \xrightarrow{\pi} M$ is called a *fibre bundle*, with (typical) fibre F, if:

$$\forall\, p \in M : \mathrm{preim}_\pi(\{p\}) \cong_{\mathrm{top}} F.$$

A fibre bundle is often represented diagrammatically as:

$$\begin{array}{ccc} F & \longrightarrow & E \\ & & \Big\downarrow \pi \\ & & M \end{array}$$

Definition. A *smooth bundle*, is a bundle (E, π, M) where E and M are smooth manifolds and the projection $\pi\colon E \to M$ is smooth. Two smooth bundles (E, π, M) and (E', π', M') are isomorphic if there exist diffeomorphisms u, f such that the following diagram commutes

$$\begin{array}{ccc} E & \xrightarrow{\quad u \quad} & E' \\ \pi \Big\downarrow & & \Big\downarrow \pi' \\ M & \xrightarrow{\quad f \quad} & M' \end{array}$$

Definition. Let G be a Lie group. A smooth bundle (E, π, M) is called a *principal G-bundle* if E is equipped with a free right G-action and

$$\begin{array}{ccc} E & & E \\ \pi\big\downarrow & \cong_{\text{bdl}} & \big\downarrow\rho \\ M & & E/G \end{array}$$

where ρ is the quotient map, defined by sending each $p \in E$ to its equivalence class (i.e. orbit) in the orbit space E/G.

Observe that since the right action of G on E is free, for each $p \in E$ we have

$$\text{preim}_\rho(G_p) = G_p \cong_{\text{diff}} G.$$

2 Resources

2.1 Written texts (not necessarily in this order)

- Overviews
  - The Road to Reality by Roger Penrose in 2004 [11]
  - Physics from Symmetry by Jakob Schwichtenberg in 2015 [13]
- General relativity
  - Catalogue of spacetimes by Mueller and Grave in 2009 [10].
  - A General Relativity Workbook by Thomas Moore in 2010 [9]
  - Gravitation by Misner, Thorne, and Wheeler in 1970 [8]
- Quantum field theory
  - Quantum Field Theory for the Gifted Amateur by Lancaster and Blundell in 2014 [6]
  - Gauge Theories in Particle Physics: A Practical Introduction, Volume 1: From Relativistic Quantum Mechanics to QED [1] and Volume 2: Non-Abelian Gauge Theories: QCD and The Electroweak Theory [2].
- Statistical physics
  - Statistical Physics of Complex Systems by Eric Bertin in 2016 [3]
  - Introduction to dynamical large deviations of Markov processes by Hugo Touchette in 2018 [15]
  - Master equations and the theory of stochastic path integrals by Weber and Frey in 2017 [16]
  - Renormalization methods: a guide for beginners by McComb in 2004 [7]
  - Effective fluctuation and response theory by Polettini and Esposito in 2019 [12]
- Global structure
  - A Visual Introduction to Differential Forms and Calculus on Manifolds by Jon-Pierre Fortney in 2018 [4].

- The geometry of physics by Theodore Frankel in 2012 [5]
- Physics and geometry by Edward Witten in 1986 [17]
- Geometry of physics by Urs Schreiber

2.2 Lectures

- Overviews
  - Research Skills and Theoretical Physics – Nima Arkani-Hamed
- General relativity
  - What is a tensor? by XylyXylyX 39 videos
  - What is a Manifold? by XylyXylyX 18 videos
  - What is General Relativity? by XylyXylyX 66 videos
  - A thorough introduction to the theory of general relativity by Frederic Schuller in 2015 28 videos
    * Lecture notes by lazierthanthou
- Quantum field theory
  - Lie Groups and Lie Algebras by XylyXylyX 41 videos
  - Quantum field theory by Tobias Osborne 18 videos
  - Advanced Quantum field theory by Tobias Osborne 18 videos
- Statistical physics
  - Statistical mechanics of particles by Mehran Kardar in 2013
  - Statistical mechanics of fields by Mehran Kardar in 2014
  - The physics of complex systems by ICTP in 2020
  - Large deviation theory in statistical physics at ICTP
    * Especially the two lectures (part 1 ; part 2) by Hugo Touchette
- Global structure
  - Lectures on Geometrical Anatomy of Theoretical Physics by Frederic Schuller in 2015
- Potential directions for future development
  - New spaces in Mathematics and Physics by the ERC Project Philosophy of Canonical Quantum Gravity

3 Evolving pedagogy

3.1 Executable physics for independent verification of understanding via concrete computations

- An important goal is to translate a minimal set of computations from textbook format to an open source computer algebra system such as Maxima in Jupyter likely making use of Viktor Toth's up-to-date implementations of atensor, ctensor, itensor (paper)

- An example of a text that approximates this goal is Functional Differential Geometry by Sussman, Wisdom, and Farr in 2013 [14]. The Catalogue of Spacetimes also contains brief computer algebra code [10].

- Edwin Woollett's reference book Maxima by Example contains examples in maxima including analysis of the Dirac equation [18].

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